ERHVERVSØKONOMI/MATEMATIK-STUDIET

Besvarelse - Statistik 2. år - 14. juni 2000

Opgave 2

1)

$$\begin{aligned} P(X_2 > 1/2) &= \int_0^1 \int_{1/2}^1 \left(\frac{3}{4}(x_1^2 + x_2^2) + \frac{1}{2}\right) dx_2 dx_1 \\ &= \int_0^1 \left(\frac{3}{4}x_1^2(\frac{1}{2}) + \frac{3}{4}[\frac{1}{3}x_2^3]_{1/2}^1 + \frac{1}{2}\cdot\frac{1}{2}\right) dx_1 \\ &= \int_0^1 \left(\frac{3}{8}x_1^2 + \frac{3}{12}(1-\frac{1}{8}) + \frac{1}{4}\right) dx_1 = \int_0^1 \frac{1}{4}(\frac{3}{2}x_1^2 + \frac{7}{8} + 1)dx_1 \\ &= \int_0^1 \frac{1}{4}(\frac{3}{2}x_1^2 + \frac{15}{8})dx_1 = \frac{1}{4}(\frac{3}{2}\cdot\frac{1}{3} + \frac{15}{8}) = \frac{1}{4}\cdot\frac{19}{8} = \frac{19}{32} \end{aligned}$$

2)

$$\begin{aligned} p(x_1) &= \int_0^1 \left(\frac{3}{4}(x_1^2 + x_2^2) + \frac{1}{2}\right) dx_2 = \frac{3}{4}x_1^2 + \frac{3}{4}\cdot\frac{1}{3} + \frac{1}{2} \\ &= \frac{3}{4}x_1^2 + \frac{3}{4} = \frac{3}{4}(x_1^2 + 1) \qquad 0 < x_1 < 1 \\ p(x_1) &= 0 \qquad\qquad\qquad ellers \end{aligned}$$

3) $Y_1 = X_1^2$

Tætheden for Y_1 er

$$q(y_1) = \frac{p(t^{-1}(y_1))}{|t'(t^{-1}(y_1))|} = \frac{\frac{3}{4}(y_1+1)}{2\sqrt{y_1}} = \frac{3}{8}(\sqrt{y_1} + \frac{\sqrt{y_1}}{y_1}) = \frac{3}{8}(\sqrt{y_1} + \frac{1}{\sqrt{y_1}})$$

idet

$$\begin{aligned} y_1 &= t(x_1) = x_1^2 \\ \frac{dy_1}{dx_1} &= 2x_1 \qquad x_1 = t^{-1}(y_1) = \sqrt{y_1} \end{aligned}$$

så

$$g(y_1) = \begin{cases} \frac{3}{8}(\sqrt{y_1} + \frac{1}{\sqrt{y_1}}) & 0 < y_1 < 1 \\ 0 & \text{ellers.} \end{cases}$$

Y_2 har samme ss.tæthed som Y_1 da der er tale om samme transformation og X_2 har samme fordeling som X_1.

4)

$$\begin{aligned} E(Y_1 \cdot Y_2) &= E(X_1^2 \cdot X_2^2) \\ &= \int_0^1 \int_0^1 x_1^2 \cdot x_2^2 (\frac{3}{4}(x_1^2 + x_2^2) + \frac{1}{2}) dx_1 dx_2 \\ &= \frac{3}{4}\left(\int_0^1 \int_0^1 x_1^4 x_2^2 dx_1 dx_2 + \int_0^1 \int_0^1 x_1^2 x_2^4 dx_1 dx_2\right) \\ &+ \frac{1}{2}\int_0^1 \int_0^1 x_1^2 x_2^2 dx_1 dx_2 \\ &= \frac{3}{4} \cdot 2 \cdot \left(\int_0^1 x_1^4 dx_1\right)\left(\int_0^1 x_2^2 dx_2\right) + \frac{1}{2}\left(\int_0^1 x_1^2 dx_1\right)\left(\int_0^1 x_2^2 dx_2\right) \\ &= \frac{3}{4} \cdot 2 \cdot \frac{1}{5} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{3} \\ &= \frac{1}{10} + \frac{1}{18} \\ &= \frac{7}{45}. \end{aligned}$$