ERHVERVSØKONOMI/MATEMATIK-STUDIET

Besvarelse reeksamen - Statistik 2. år - 4. august 2000

Opgave 2

1)

$$\begin{aligned} EX_1 &= \int_0^B \theta B^{-\theta} x^{\theta-1} \cdot x \, dx = \theta B^{-\theta} \int_0^B x^\theta \, dx = \frac{\theta}{1+\theta} B \\ EX_1^2 &= \theta B^{-\theta} \int_0^B x^{\theta+1} dx = \frac{\theta}{\theta+2} B^{-\theta} B^{\theta+2} = \frac{\theta}{\theta+2} B^2 \end{aligned}$$

så

$$\begin{aligned} Var(X_1) = \frac{\theta}{\theta+2}B^2 - \frac{\theta^2}{(1+\theta)^2}B^2 &= \theta B^2 \left(\frac{1}{\theta+2} - \frac{\theta}{(1+\theta)^2} \right) \\ &= \theta B^2/(\theta+2)(1+\theta)^2 \end{aligned}$$

2) $Y = \frac{X_1}{B}$. Lad q betegne tætheden for Y. Eksemplet i TT om skalatransformation p86 giver direkte

$$q(y) = Bp(yB) = B\theta B^{-\theta}(yB)^{\theta-1} = \theta y^{\theta-1}$$

så

$$q(y) = \begin{cases} \theta y^{\theta-1} & 0 < y \leq 1 \\ 0 & \text{ellers} \end{cases}$$

3)

$$P(V \leq v) = P(max\{X_1, \ldots, X_n\} \leq v) = (P(X_1 \leq v))^n$$

pga. uafhængigheden af $X_1, X_2, \ldots, X_n$

Fordelingsunktionen for X_1 er

$$P(X_1 \leq v) = \int_0^v \theta B^{-\theta} x^{\theta-1} dx = \theta B^{-\theta} \left[\frac{1}{\theta} x^{\theta}\right]_0^v$$
$$= B^{-\theta} v^{\theta} \quad 0 < v \leq B$$

så

$$P(V \leq v) = \begin{cases} 0 & v \leq 0 \\ B^{-n\theta} v^{n\theta} & 0 < v \leq B \\ 1 & v > B \end{cases}$$