Opgave 1, reeksamen august 2007.

(a)

$$P(X_1 + X_2 \leq 2)$$
$$= P((X_1, X_2) \in \{(0,0), (0,1), (0,2), (1,0), (1,1), (2,0)\})$$
$$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{8} + \frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{4} + \frac{1}{8} \times \frac{1}{2}$$
$$= \frac{4 + 2 + 1 + 2 + 1 + 1}{16} = \frac{11}{16} = \mathbf{0.6875}$$

(b)

$$\mathrm{E}(Y) = \sum_{x=0}^{\infty} \left(\frac{1}{3}\right)^x \left(\frac{1}{2}\right)^{x+1} = \frac{1}{2} \sum_{x=0}^{\infty} \left(\frac{1}{6}\right)^x$$
$$= \frac{1}{2} \times \frac{1}{1 - \frac{1}{6}} = \frac{1}{2} \times \frac{6}{5} = \frac{3}{5} = \mathbf{0.6000}$$

(c)

$$P(X_1 \leq 3 \mid X_1 + X_2 = 5) = \frac{P(X_1 \leq 3 \text{ og } X_1 + X_2 = 5)}{P(X_1 + X_2 = 5)}$$
$$= \frac{P((X_1, X_2) \in \{(0,5), (1,4), (2,3), (3,2)\})}{P((X_1, X_2) \in \{(0,5), (1,4), (2,3), (3,2), (4,1), (5,0)\})}$$
$$= \frac{4 \times \left(\frac{1}{2}\right)^7}{6 \times \left(\frac{1}{2}\right)^7} = \frac{4}{6} = \frac{2}{3} = \mathbf{0.6667}$$