Opgave 1, eksamen juni 2008.

(a)

$$P(X = 2 \text{ og } Y \leq 2) = P((X, Y) \in \{(2, 1), (2, 2)\}) = 0.2 + 0.1 = \mathbf{0.3}\,.$$

(b) Hvis q betegner sandsynlighedsfunktionen for Y er

$q(1) = P(Y = 1) = 0.1 + 0.2 = \mathbf{0.3}$,
$q(2) = P(Y = 2) = 0.2 + 0.1 = \mathbf{0.3}$,
$q(3) = P(Y = 3) = 0.2 + 0.2 = \mathbf{0.4}$.

Middelværdi og varians for Y:

$\mathrm{E}(Y) = 0.3 \times 1 + 0.3 \times 2 + 0.4 \times 3 = \mathbf{2.1}$.

$\mathrm{E}(Y^2) = 0.3 \times 1 + 0.3 \times 4 + 0.4 \times 9 = 5.1$,

$\mathrm{var}(Y) = 5.1 - 2.1^2 = \mathbf{0.69}$.

(c) Hvis q betegner sandsynlighedsfunktionen for den betingede fordeling af Y, givet $X = 2$, er

$q(1) = P(Y = 1 \mid X = 2) = 0.2/0.5 = \mathbf{0.4}$,
$q(2) = P(Y = 2 \mid X = 2) = 0.1/0.5 = \mathbf{0.2}$,
$q(3) = P(Y = 3 \mid X = 2) = 0.2/0.5 = \mathbf{0.4}$.

Middelværdi og varians for Y, givet $X = 2$:

$\mathrm{E}(Y \mid X = 2) = 0.4 \times 1 + 0.2 \times 2 + 0.4 \times 3 = \mathbf{2.0}$.

$\mathrm{E}(Y^2 \mid X = 2) = 0.4 \times 1 + 0.2 \times 4 + 0.4 \times 9 = 4.8$,

$\mathrm{var}(Y \mid X = 2) = 4.8 - 2^2 = \mathbf{0.80}$.