Facitliste

Facitlisten giver slutresultater for opgaver, der har en egentlig løsning. Opgaver af typen "Bevis ... ", "Forklar ... ", etc. er altså ikke medtaget.

Kapitel 1: Sandsynlighedsfordelinger og stokastiske variable

Opgave 1.2.1. (a) $\frac{6}{36} = 0.1667$
(b) $\frac{6}{36} = 0.1667$
(c) $\frac{11}{36} = 0.3056$
(d) $\frac{10}{36} = 0.2778$

Opgave 1.2.2. $\frac{27}{216} = 0.1250$

Opgave 1.2.3. (a) $\frac{1}{365} = 0.0027$
(b) $\frac{364}{365} = 0.9973$

Opgave 1.2.4. (a) $\frac{1}{7^7} = 0.00000121$
(b) $\frac{1}{7^6} = 0.00000850$
(c) $\frac{7!}{7^7} = 0.00611990$

Opgave 1.2.5. $\frac{36^{(13)}}{52^{(13)}} = 0.003639$

Opgave 1.3.1.

1	1.0000	11	0.8589	21	0.5563
2	0.9973	12	0.8330	22	0.5243
3	0.9918	13	0.8056	23	0.4927
4	0.9836	14	0.7769	24	0.4617
5	0.9729	15	0.7471	25	0.4313
6	0.9595	16	0.7164	26	0.4018
7	0.9438	17	0.6850	27	0.3731
8	0.9257	18	0.6531	28	0.3455
9	0.9054	19	0.6209	29	0.3190
10	0.8831	20	0.5886	30	0.2937

Opgave 1.3.2. $1 - (\frac{5}{6})^5 = 0.5981$

Opgave 1.3.4. $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} - \frac{1}{36} - \frac{1}{36} - \frac{1}{36} + \frac{1}{36} = \frac{16}{36} = 0.4444$

Opgave 1.4.1.

y	q(y)
1	0.40
2	0.11
3	0.33
4	0.16

Opgave 1.4.2.

z1	P(Z1=z1)
1	11/36 = 0.3056

```
2      9/36 =  0.2500
3      7/36 =  0.1944
4      5/36 =  0.1389
5      3/36 =  0.0833
6      1/36 =  0.0278
```

OPGAVE 1.4.3.

```
y         q(y)
0      1/16 = 0.0625
1      4/16 = 0.2500
2      6/16 = 0.3750
3      4/16 = 0.2500
4      1/16 = 0.0625
```

OPGAVE 1.4.4.

```
y           q(y)
3       1/216 = 0.0046
4       3/216 = 0.0139
5       6/216 = 0.0278
6      10/216 = 0.0463
7      15/216 = 0.0694
8      21/216 = 0.0972
9      25/216 = 0.1157
10     27/216 = 0.1250
11     27/216 = 0.1250
12     25/216 = 0.1157
13     21/216 = 0.0972
14     15/216 = 0.0694
15     10/216 = 0.0463
16      6/216 = 0.0278
17      3/216 = 0.0139
18      1/216 = 0.0046
```

Se tegning side 146.

OPGAVE 1.4.5.

```
y            q(y)
0      216/343 = 0.6297
1      108/343 = 0.3149
2       18/343 = 0.0525
3        1/343 = 0.0029
```

OPGAVE 1.4.6. Mængderne $t^{-1}(y)$, $y \in F$, skal være lige store.

Kapitel 2: Betingede fordelinger og uafhængighed

OPGAVE 2.1.1. (a) $\frac{1}{3} = 0.3333$
(b) 0
(c) $\frac{1}{6} = 0.1667$
(d) $\frac{7}{25} = 0.2800$

OPGAVE 2.1.2. (a) 0.30
(b) $\frac{0.40}{0.51} = 0.7843$

OPGAVE 2.1.3. (a) $\frac{1}{2} = 0.5000$
(b) $\frac{1}{10} = 0.1000$

OPGAVE 2.1.4. $\frac{334^{(25)}}{334^{25}} = 0.3981$

OPGAVE 2.2.1. (a) $P(X_1 = x \mid Y = 7) = \frac{1}{6}$ for alle $x \in \{1, \ldots, 6\}$
(b) $P(Z_1 = z_1 \mid Y = 7) = \frac{1}{3}$ for $z_1 \in \{1, 2, 3\}, 0$ ellers
(c):

$$P(Z_2 = z_2 \mid X_1 = 3) = \begin{cases} 0 \text{ for } z_2 \in \{1, 2\} \\ \frac{1}{2} \text{ for } z_2 = 3 \\ \frac{1}{6} \text{ for } z_2 \in \{4, 5, 6\} \end{cases}$$

(d) Se tegning side 146.

OPGAVE 2.2.2. (a) $P(X_1 = 0 \mid X_1 + \cdots + X_{10} = 7) = \frac{3}{10}$
(b) Som i opgave 1.4.3.

OPGAVE 2.3.1.

$$P(X = x) = \begin{cases} \frac{1}{2^x} \text{ for } x \in \{1, \ldots, 10\} \\ \frac{1}{2^{10}} \text{ for } x = 11 \end{cases}$$

OPGAVE 2.3.2.

$$P(X = x) = \begin{cases} \frac{1}{6}(\frac{5}{6})^{x-1} \text{ for } x \in \{1, \ldots, 99\} \\ (\frac{5}{6})^{99} \text{ for } x = 100 \end{cases}$$

OPGAVE 2.3.3. $P(x \text{ kugler tilbage}) = \frac{10^{(11-x)}}{12^{(11-x)}} \frac{2}{x+1} = \frac{x}{66}, x \in \{1, \ldots, 11\}$

OPGAVE 2.3.4. 0.2297

OPGAVE 2.4.5. $\exists y \in F : P\left(X \in t^{-1}(y)\right) = 1$

Kapitel 3: Fordelinger af antal

Opgave 3.1.2. (a) $\binom{52}{13} = 635013559600$
(b) $4/\binom{52}{13} = 0.0000000000063 = 6.3 \times 10^{-12}$

Opgave 3.2.2. (a) 0.0322
(b) 0.0355
(c) 0.1929
(d) 0.2130

Opgave 3.2.3. (a) $n - S \sim \text{bin}(n, 1-p)$
(b) $S_1 + S_2 \sim \text{bin}(n_1 + n_2, p)$

Opgave 3.3.2.

r	Hyp.Geo.	Binomial
0	0.0020	0.0028
1	0.0237	0.0278
2	0.1117	0.1159
3	0.2648	0.2575
4	0.3334	0.3219
5	0.2115	0.2146
6	0.0529	0.0596

Opgave 3.3.4. $E = \{0 \vee (n-H), \ldots, R \wedge n\}$

Opgave 3.3.7. (a) 0.2573
(b) 0.2936
(c) Hypergeometrisk $N = 48$, $R = 4$, $n = 13$:

antal	ssh.
0	0.2691
1	0.4373
2	0.2385
3	0.0514
4	0.0037

(d) Nej

Opgave 3.4.1. (a) 0.003438
(b) 0.08942

Opgave 3.6.1. (a) $e^{-1} = 0.3679$
(b) $e^{-3} = 0.0498$
(c) $1 - e^{-1/3} = 0.2835$

Opgave 3.7.1. Se tegning side 146.

Opgave 3.7.3. Ventetiden – 2 er negativt binomialfordelt med $n = 3$ og $p = 1 - e^{-1/30}$

Kapitel 4: Middelværdi og varians

OPGAVE 4.2.5. $n\frac{1-p}{p}$

OPGAVE 4.3.1. (a) m.v. $= p$, var. $= p(1-p)$, std.afv. $= \sqrt{p(1-p)}$
(b) m.v. $= 3.5$, var. $= 2.9167$, std.afv. $= 1.7078$
(c) m.v. $= 7$, var. $= 5.8333$, std.afv. $= 2.4152$
(d) m.v. $= \frac{1}{2}(n+1)$, var. $= \frac{1}{12}(n^2-1)$,
std.afv. $= \sqrt{\frac{1}{12}(n^2-1)}$

OPGAVE 4.3.4. m.v. $= 1000$, var. $= 1000$

OPGAVE 4.3.6. 74

OPGAVE 4.3.7. m.v. $= 12$, var. $= 132$

OPGAVE 4.3.8. e^{λ}

OPGAVE 4.4.1.

$$\frac{-p_i p_j}{\sqrt{(p_i - p_i^2)(p_j - p_j^2)}} = -\sqrt{\frac{p_i}{1-p_j}\frac{p_j}{1-p_i}}$$

($= -1$ hvis og kun hvis $p_i + p_j = 1$)

OPGAVE 4.5.1. Chebychev's ulighed giver specielt

$$P\left(|Y - \mathrm{E}Y| < 4.472\sqrt{\mathrm{var}(Y)}\right) \geq 0.95$$

$$P\left(|Y - \mathrm{E}Y| < 2\sqrt{\mathrm{var}(Y)}\right) \geq 0.75$$

($4.472 = \sqrt{20}$)

Kapitel 5: Fordelinger på den reelle akse

OPGAVE 5.4.1. (a) $2yp(y^2)$, $y > 0$
(b) $\frac{1}{y^2}p(\frac{1}{y})$, $y \neq 0$
(c) $\frac{1}{2\sqrt{y}}\left(p(\sqrt{y}) + p(-\sqrt{y})\right)$, $y > 0$
(d) $\frac{1}{y}p(\log y)$, $y > 0$

OPGAVE 5.4.2. Fordelingsfunktion for X, givet $X \in [a,b]$:

$$F^{[a,b]}(x) = \begin{cases} 0 \text{ for } x \leq a \\ \frac{F(x)-F(a)}{F(b)-F(a)} \text{ for } a < x < b \\ 1 \text{ for } x \geq b \end{cases}$$

OPGAVE 5.4.4. (a) $F(y) = \frac{1}{\pi}\mathrm{Arctan}(y) + \frac{1}{2}$
(b) $Y = |X|$: $\frac{2}{\pi(1+y^2)}$, $y > 0$
$Y = X^2$: $\frac{1}{\pi(1+y)\sqrt{y}}$, $y > 0$
$Y = 1/X$: $\frac{1}{\pi(1+y^2)}$

Opgave 5.4.5. (a) 0.0513, 0.6931 og 2.9957
(b) -6.3138, 0 og 6.3138

Opgave 5.4.6. $p(y) + p(-y)$, $y \geq 0$

Opgave 5.4.7. Tæthed:

$$\frac{1}{\pi\sqrt{r^2 - (x-r)^2}}\ ,\ x \in]0, 2r[$$

hvor r betegner hjulets radius.

Opgave 5.5.1. m.v. $= 0$, var. $= \frac{1}{2}$. Se tegning side 147.

Opgave 5.5.2. m.v. $= 0$, var. $= 2$. Se tegning side 147.

Opgave 5.5.3. (a) 6
(b) $\frac{1}{1+a}$ for $a > -1$

Opgave 5.5.4. (b) m.v. $= n + 1$, var. $= = n + 1$, std.afv. $= \sqrt{n+1}$.

Opgave 5.6.1. Se tegning side 147.

Opgave 5.6.2. Se tegning side 148.

Opgave 5.6.3. Mellem 0.0008 og 0.0010, afhængigt af beregningsmåde.

Opgave 5.6.4. (a) $\frac{1}{y\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\log y - \mu)^2}{2\sigma^2}\right)$, $y > 0$
(b) Y log–norm. $(\mu, \sigma^2) \Rightarrow \beta Y$ log–norm. $(\mu + \log \beta, \sigma^2)$.

Kapitel 6: Fordelinger på reelle talrum

Opgave 6.2.3. (b) $30x^2h(1-x-h)^2$
(c) $30x^2(1-x)^2$

Opgave 6.2.5. Y's tæthed: $\frac{1}{(y+1)^2}$, $y > 0$
S's tæthed: se^{-s}, $s > 0$

Opgave 6.3.1.

(a)

$$q(y) = \begin{cases} y \text{ for } 0 \leq y \leq 1 \\ 2 - y \text{ for } 1 < y \leq 2 \end{cases} \qquad \text{(0 ellers)}$$

(b)

$$q(y) = \begin{cases} \frac{1}{2}y^2 \text{ for } 0 \leq y \leq 1 \\ 3y - y^2 - 1.5 \text{ for } 1 < y \leq 2 \\ \frac{1}{2}y^2 - 3y + 4.5 \text{ for } 2 < y \leq 3 \end{cases} \qquad \text{(0 ellers)}$$

(c)

$$q(y) = \begin{cases} \frac{1}{10}y \text{ for } 0 \leq y \leq 1 \\ \frac{1}{10} \text{ for } 1 < y \leq 10 \\ \frac{1}{10}(11 - y) \text{ for } 10 < y \leq 11 \end{cases} \qquad \text{(0 ellers)}$$

Se tegning side 148.

OPGAVE 6.3.2. Tosidet eksponentialfordeling — se opgave 5.5.2

OPGAVE 6.3.4. (a) $2e^{-2m}$, $m > 0$
(b) $2e^{-m-s}$, $0 < m < s$
(c) $2e^{-2m-d}$, $m, d > 0$
(d) $2(e^{-s} - e^{-2s})$, $s > 0$

OPGAVE 6.3.5. (a) Tæthed $\frac{1}{r_1}$, $0 < z < r_1$
(b) Tæthed $-\log z$, $0 < z < 1$
(c) X_1 og X_2 er uafh., normeret eksponentialfordelte
(d) Tæthed se^{-s}, $s > 0$
(e) ...

OPGAVE 6.3.6.

$$q(s) = \begin{cases} 1 - e^{-s} \text{ for } 0 \le s \le 1 \\ (e-1)e^{-s} \text{ for } s > 1 \end{cases} \qquad \text{(0 ellers)}$$

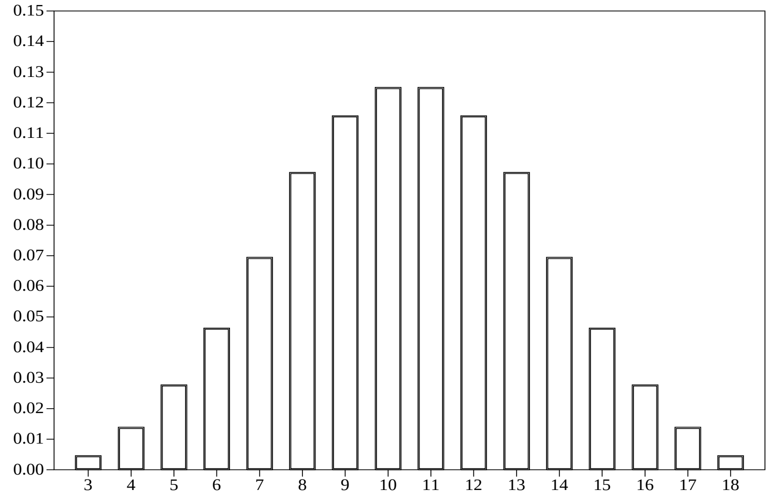
Se tegning side 148.

Kapitel 7: Den normale fordelings teori

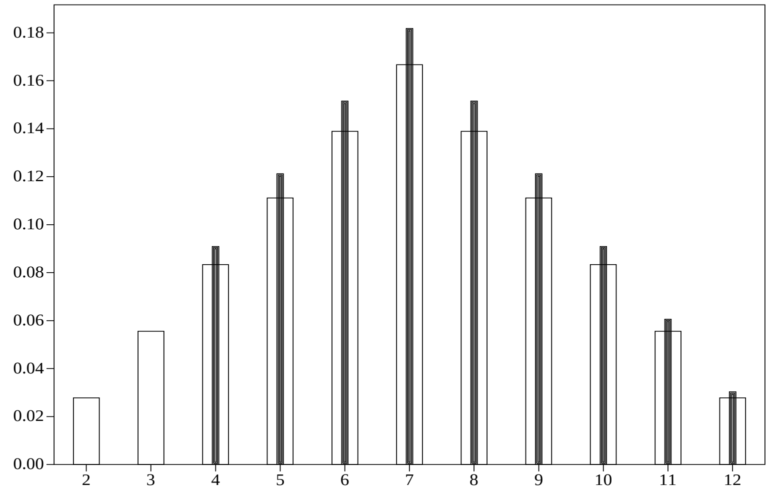
OPGAVE 7.1.1. Middelværdi n, varians $2n$. Se tegning side 149.

OPGAVE 7.1.2. Se tegning side 149.

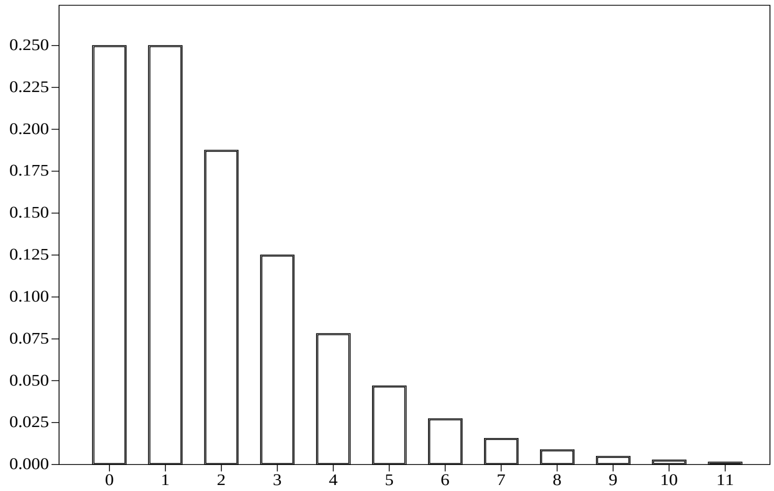
Opgave 1.4.4



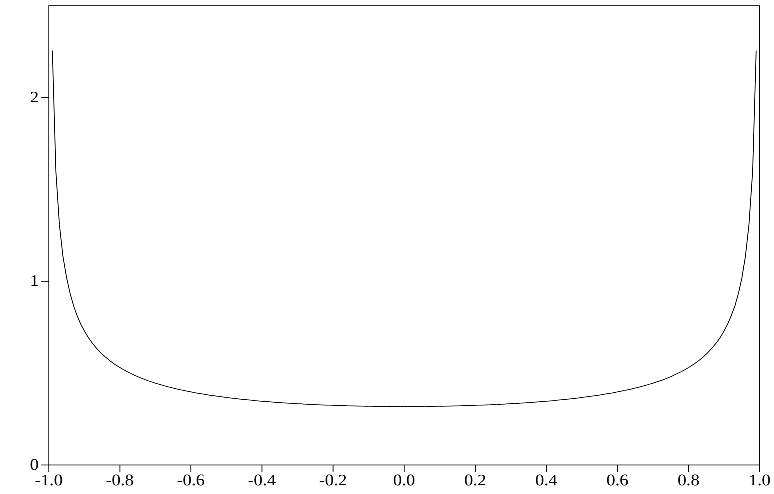
Opgave 2.2.1



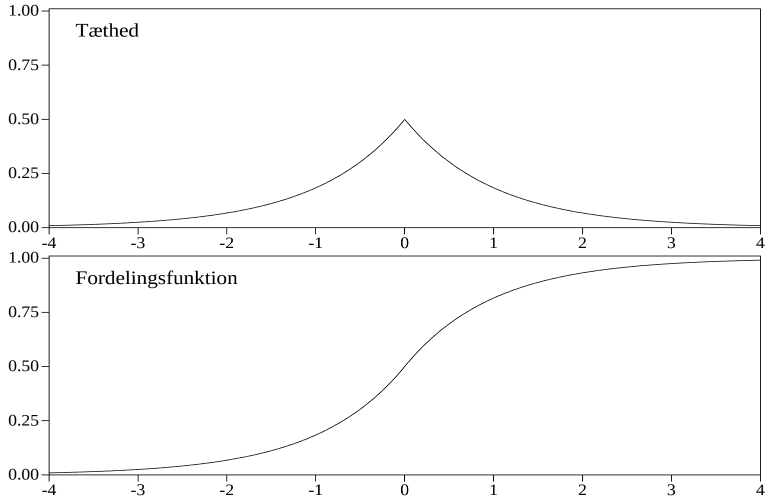
Opgave 3.7.1



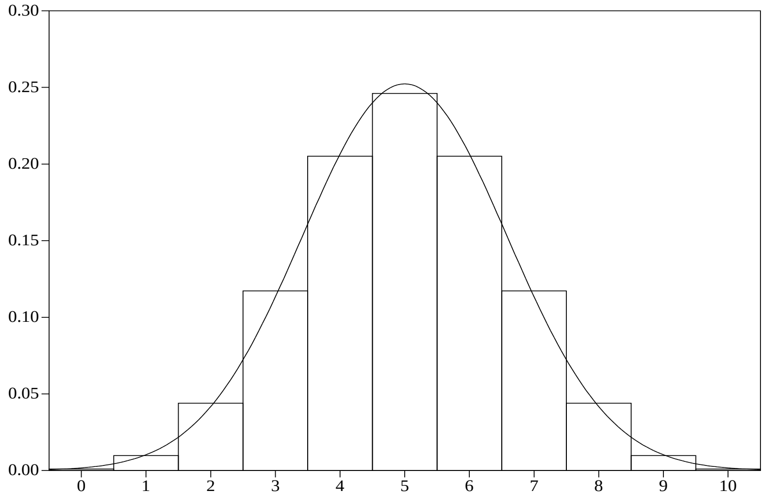
Opgave 5.5.1



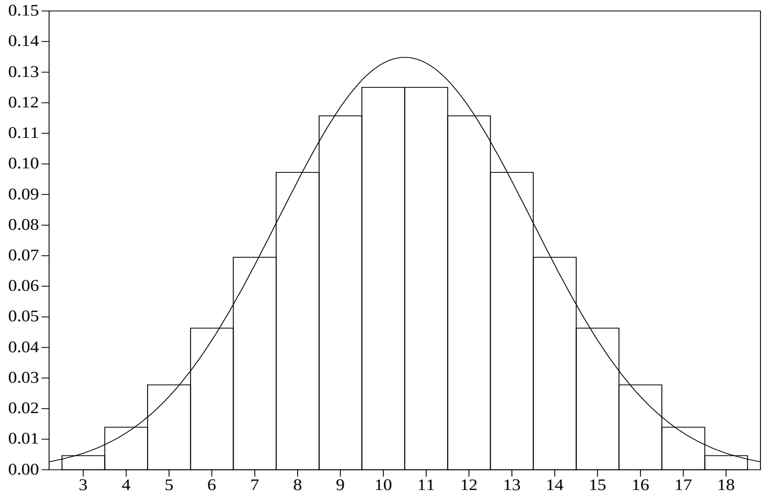
Opgave 5.5.2



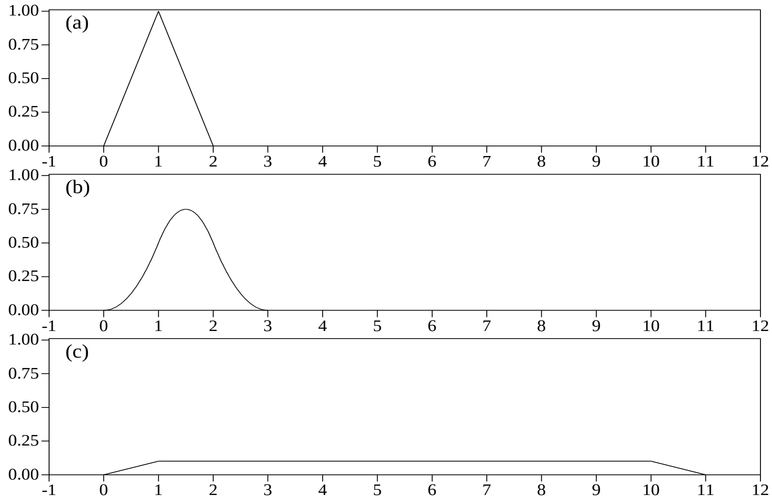
Opgave 5.6.1



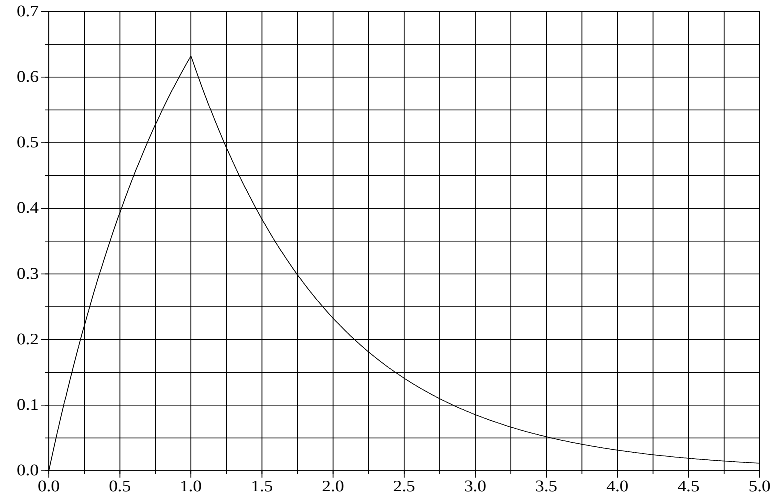
Opgave 5.6.2



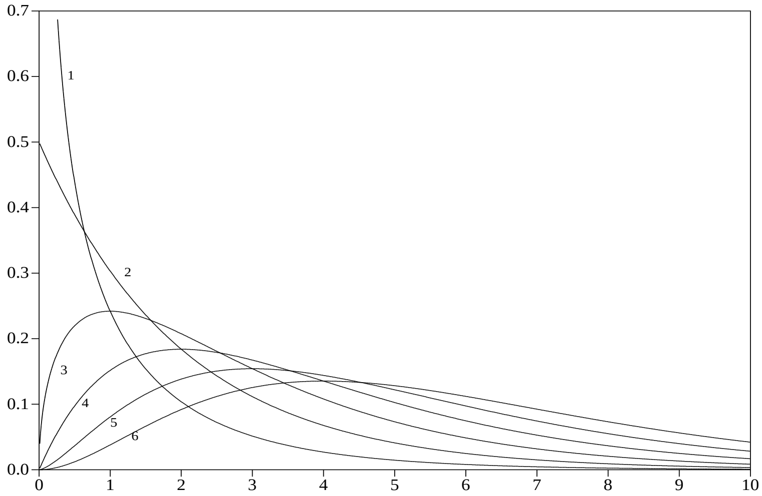
Opgave 6.3.1



Opgave 6.3.6



Opgave 7.1.1



Opgave 7.1.2